**Lecture 9.**

**Differentials. Properties of differential. Derivatives of Higher Orders. Leibniz formula. Higher-order differentials.**

**Definition.** The **differential** (first order) of a function  is the principal part of its increment, which part is linear relative to the increment  of the independent variable . The differential of a function is equal to the product of its derivative by the differential of the independent variable

, (1)

hence

. (2)

**Example.** Find the increment and differential of a function

**Solution:**

*First method*:

or

.

Hence,

*Second method:*

By (1) we get

**Definition of higher derivatives.** A derivative of the second order, or *second derivative*, of the function is the derivative of its derivative; that is, The second derivative may be denoted as

If is the law of rectilinear motion of a point, then is the acceleration of this motion.

Generally, the *n-*th *derivative* of a function is the derivative of a derivative of order (*n-1).* For the n-th derivative we use the notation

**Leibniz rule.** If the functions have derivatives up to the *n-*th order inclusive, then to evaluate the *n-*th derivative of a product of these functions we can use Leibniz rule (or formula):



Since,



then we have



consequently, we can write Leibniz formula in the form



**Higher order differentials:** A second order differentialis the differential of a first order differential:

(10)

We similarly define the differentials of the third and higher orders.

If and is an independent variable, then

